



- Thermophysical properties, measurements and uncertainties

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- ❑ Introduction
- ❑ Thermodynamic properties and experimental methods
- ❑ Transport properties and experimental methods
- ❑ Uncertainties
- ❑ Validation of data / Data treatment
  - Pure compounds
  - Mixtures
- ❑ Conclusion

□ Availability of accurate thermophysical properties is

important for:

- Design and construction of new process plant equipments
- Implementation of “more rigorous and sophisticated design procedures” in software packages (Raal and Mühlbauer, 1984)

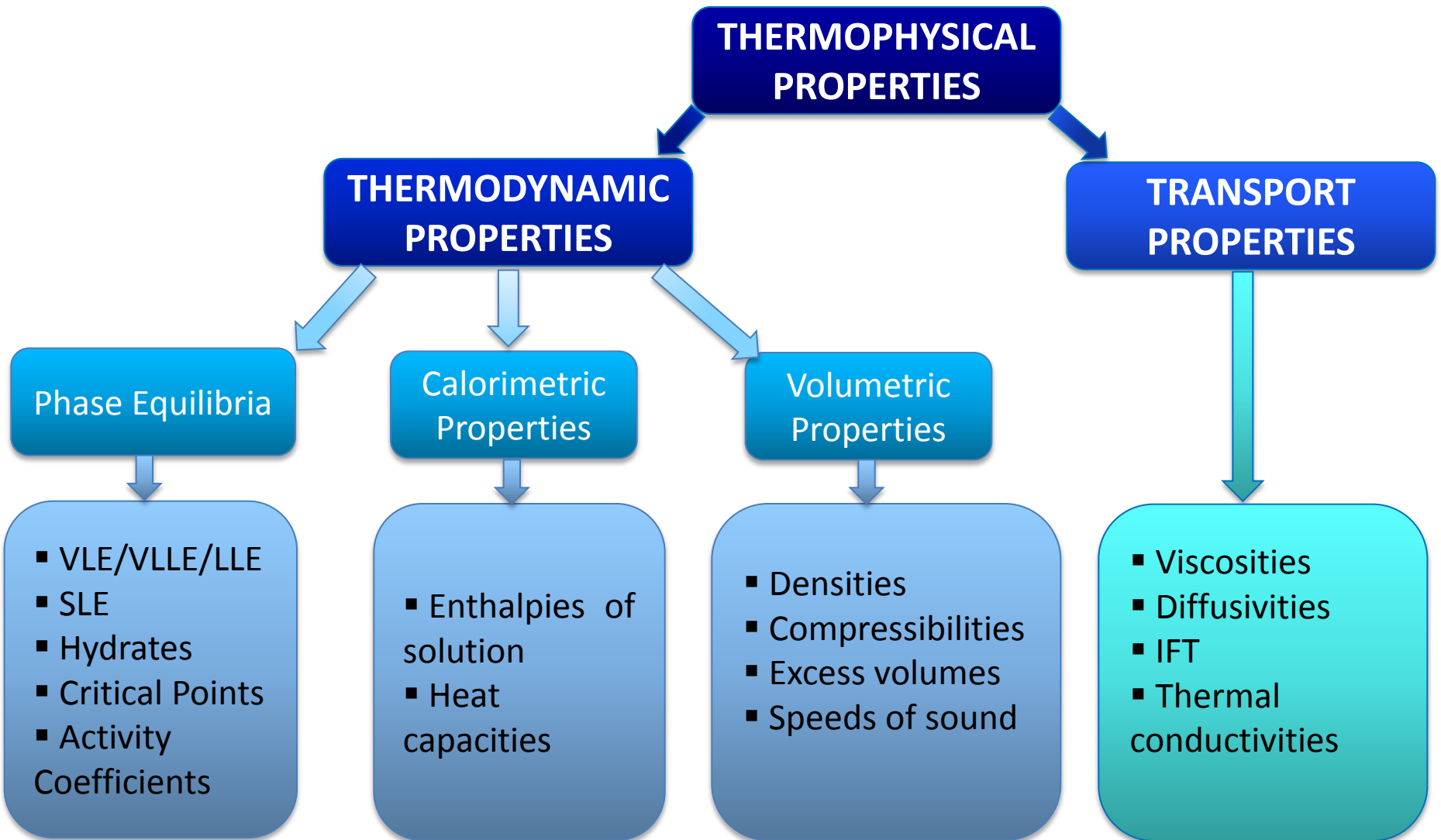
- ❑ Thermophysical properties may be divided into two groups:
  - Thermodynamic properties
  - Transport properties
- ❑ For each experimental measurement, there is:
  - An experimental setup
  - An experimental procedure
- ❑ For each data measured, we get:
  - A numerical value
  - A unit
  - Its uncertainty
- ❑ Validation of data and/or data treatment

## ❑ What can we measure?

- Temperature
- Pressure
- Composition
- Volume (variation of volume) or density
- Speed of sound
- Flow
- Luminosity
- Hardness
- Etc

## ❑ Aim: disturb the system as little as possible while being accurate and reliable

- Research on techniques and experimental procedures
- Size of the equilibrium cell
- Development of sensors



## □ The choice of the adequate experimental method

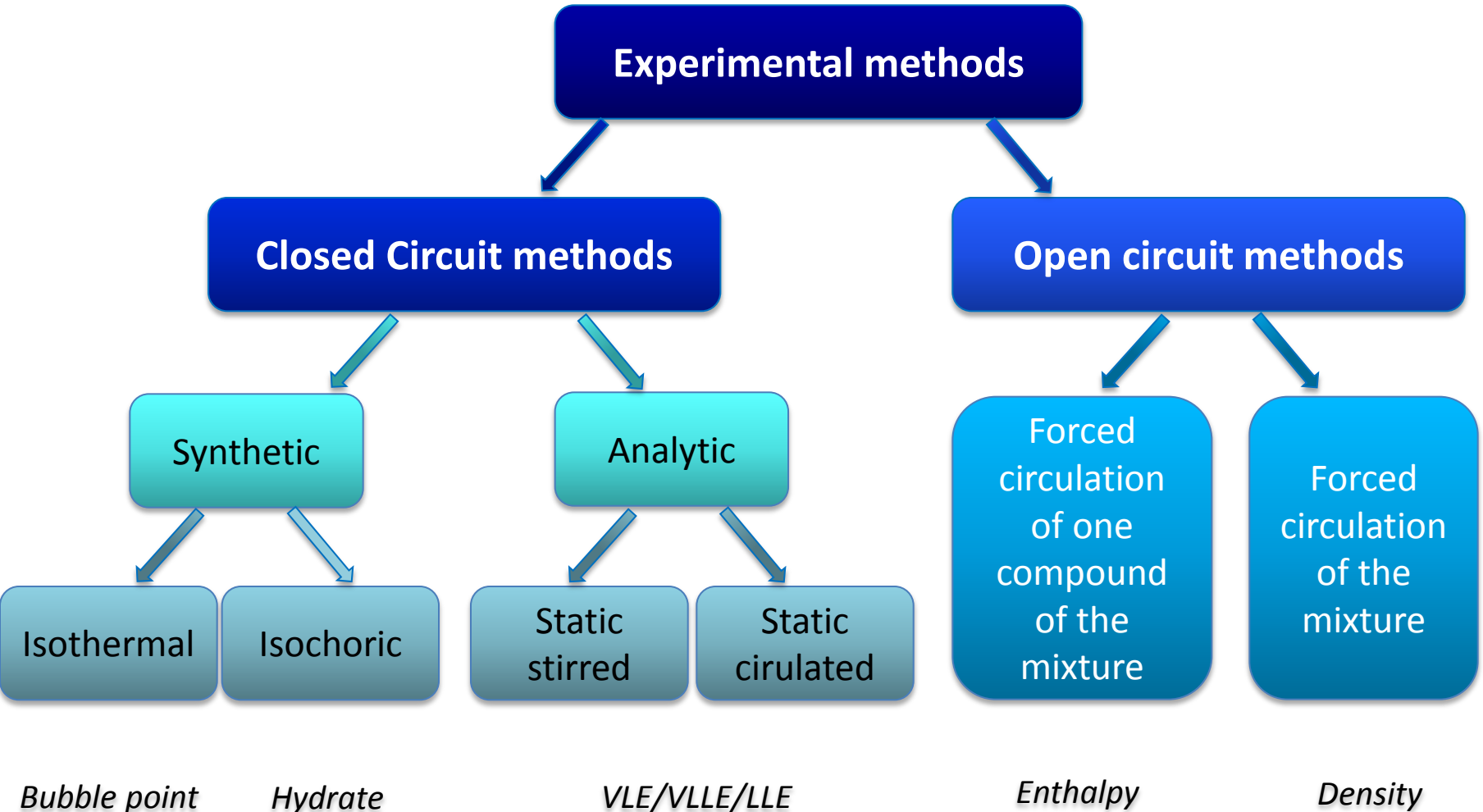
depends on:

- The thermophysical property ( $P$ Tx, density, activity coefficient...)
- The type of fluid (molar mass, toxicity, stability within temperature...)
- Temperature(s) and pressure(s) of interest



# Thermodynamic properties and Experimental methods



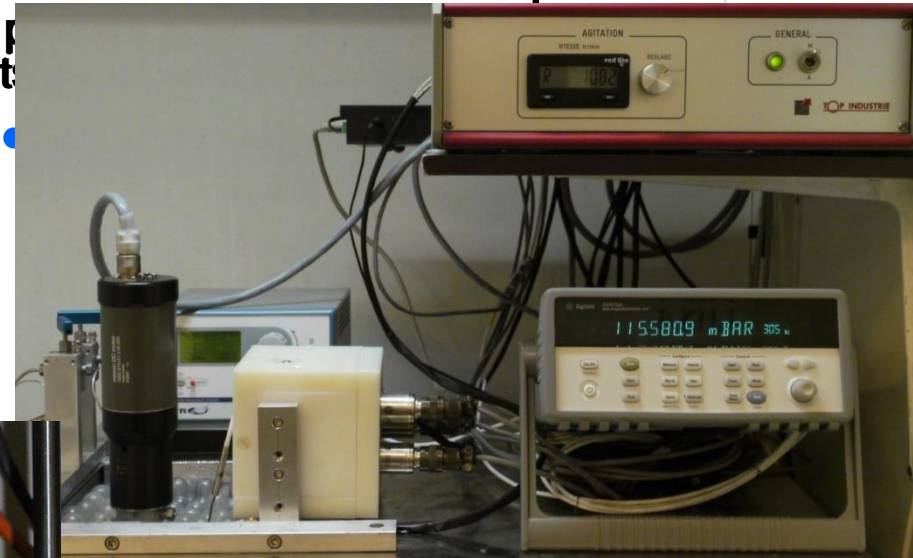
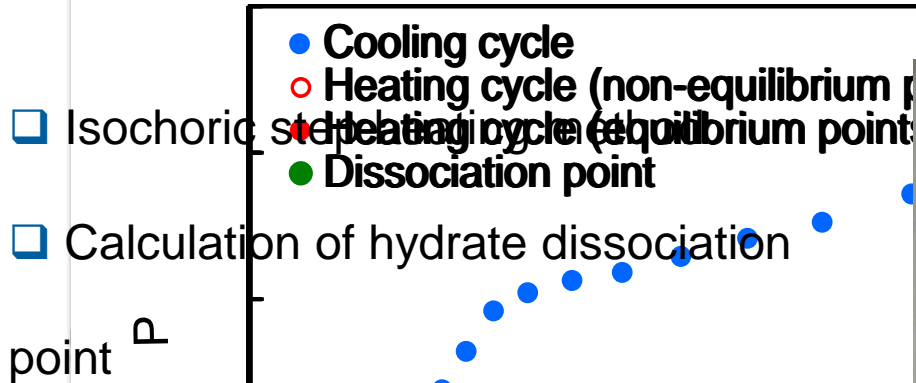


- ❑ Static analytic method
- ❑ Vapour and/or liquid sampling (ROLSI™)
- ❑ Gas chromatography
- ❑ Data obtained: composition of each phase



*VLE Equipment*  
*Up to 40 MPa / [300 – 470] K*

# Determination of hydrate dissociation point



*Equipment for hydrate dissociation points measurements*

*Up to 70 MPa / [200 – 320] K*

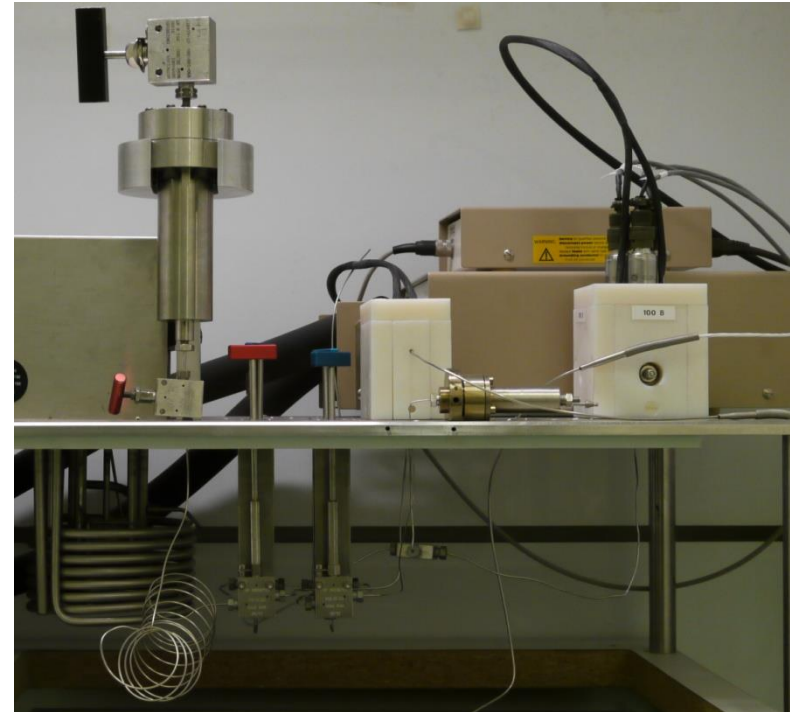
## ❑ Indirect synthetic method

## ❑ Vibrating tube densimeter:

- Relation between the vibrating period of a dimensional resonator and its vibrating mass

$$\rho = \frac{m_0}{V_{\text{int}}} \left[ \left( \frac{K\tau^2}{K_0\tau_0^2} \right) - 1 \right]$$

Period of vibration of the tube  
K and  $\tau$  are functions of the tube  
material and geometry  
[kg.m<sup>-3</sup>.s<sup>-2</sup>]



*Vibrating tube densimeter*  
Up to 70 MPa / [250 – 390] K

- ❑ Open circuit method
- ❑ T and P are constant
- ❑ Experimental procedure:
  - T fixed
  - Fluids are loaded into the pumps
  - Fluids are pressurized
  - Circulation at different flow rates ( $x_i \in [0 ; 1]$ )



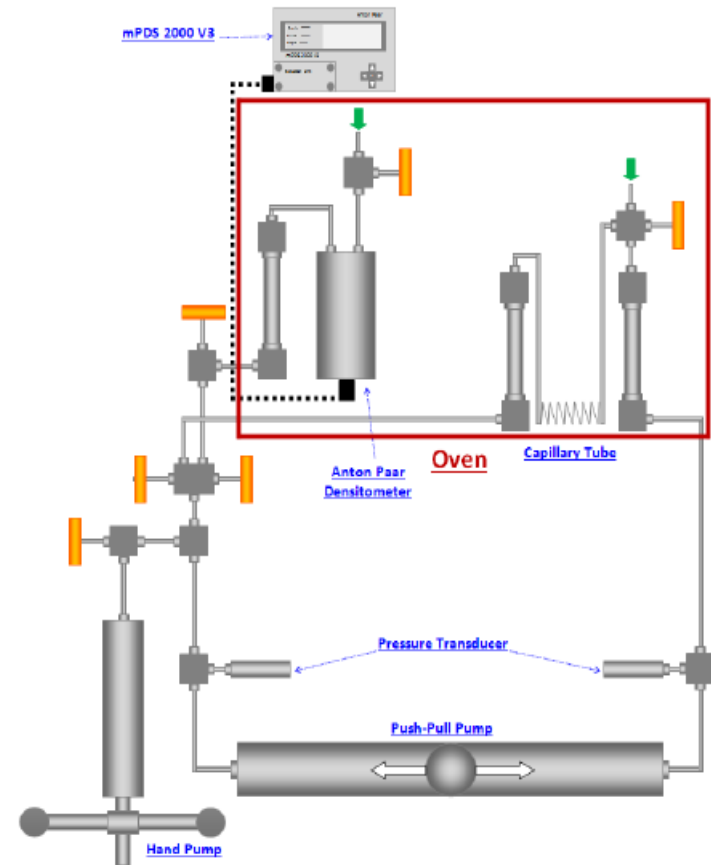
*Calorimeter*



# Transport properties and Experimental methods

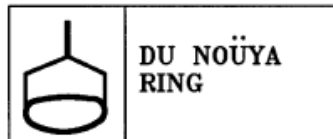
- ❑ Capillary tube viscosity measurement method
- ❑ Consistency of measurements ensured with specific experimental procedure:
  - At each pressure, viscosities determined at different flow rates
  - Laminar flow conditions ensured: check Reynolds number  $Re = \frac{\rho V L}{\mu} < 2300$
- ❑ Pumping the sample fluid through capillary tube by the piston pump results in dynamic differential pressure (monitored and recorded)
- ❑ Poiseuille equation can relate the pressure drop across the capillary tube to the viscosity

$$\Delta P = \frac{128 L Q \eta}{\pi C D^4}$$

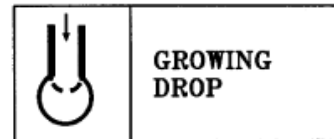
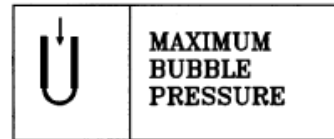


## □ Different methods:

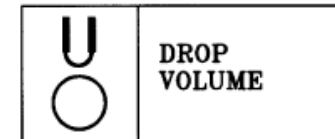
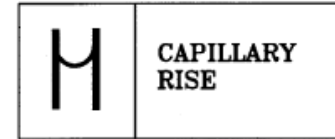
### I. Direct Measurement Using a Microbalance



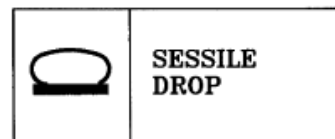
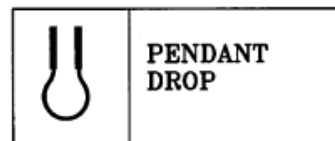
### II. Measurement of Capillary Pressure



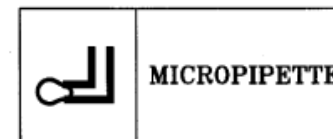
### III. Analysis of Capillary- Gravity Forces



### IV. Gravity-Distorted Drops



### V. Reinforced Distortion of Drop



Source:  
Drelich et al.,  
Encyclopedia  
of Surface  
and Colloid  
Science,  
2002





# Uncertainties

## □ Uncertainty:

- « **Quantitative indication** of the **quality** of a result »
- « **Doubt** that one has on a result. It corresponds to a **dispersion** that can be reasonably attributed to the **measurement process** »

## □ Estimation:

- Based on NIST procedure
- Type A: calculation based on statistical analysis of a set of observations
- Type B: evaluation by other means:
  - ❖ Ex: distribution law
  - ❖ Calibration

## ❑ Calibration:

process of finding a **relationship** between the physical property and the output signal

## ❑ Required:

- New instrument
- After an instrument has been repaired or modified
- Before and/or after measurement
- After an event (shock, sudden shutdown, etc)

## ❑ Calibration and errors:

process = (measure – standard measure) + errors

- ✓ Material
- ✓ Equipment
- ✓ Method
- ✓ Operator
- ✓ Environment

## □ Uncertainty related to repeatability: type A

➤ Ex:

$$u_{\text{rep}}(P) = \sqrt{\frac{1}{N(N-1)} \sum_{k=1}^N (P_k - \bar{P})^2}$$

## □ Type B and distribution laws

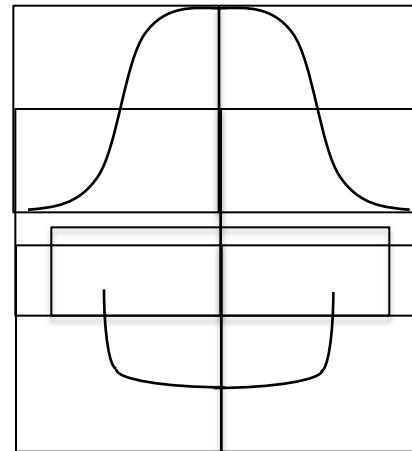
➤ The uncertainty is supposed to be  $\pm a$

➤ Different distribution laws:

❖ Normal distribution  $u_{\text{ref}} = \frac{a}{2}$

❖ Uniform distribution  $u_{\text{ref}} = \frac{a}{\sqrt{3}}$

❖ Arcsine distribution  $u_{\text{ref}} = \frac{a}{\sqrt{2}}$



# Different sources of uncertainties

## ○ Exemple: VLE / LLE

	Repeatability	Calibration		
$u(P)$	$u_{rep}(P)$	$u_{ref}(P)$	$u_{corr}(P)$	Polynomial correlation to convert the « read » value to the « measured » value
$u(T)$	$u_{rep}(T)$	$u_{ref}(T)$	$u_{corr}(T)$	
$u(x_i)$	$u_{rep}(x_i)$	$u_{inj}(n_j)$	$u_{corr}(n_j)$	$u_{rep}(S_j)$

Standard deviation

Standard deviation

Calibration

Polynomial correlation

Repeatability of surfaces at fixed volume

- N volumes are injected leading to N surfaces
- polynomial correlation between volume and surfaces
- polynomial correlation between the number of moles and surfaces

## □ Molar fraction

$$\text{➤ } u(x_1) = \sqrt{u_{\text{rep}}^2(x_1) + \left(\frac{1-x_1}{n_1+n_2}\right)^2 u^2(n_1) + \left(\frac{x_1}{n_1+n_2}\right)^2 u^2(n_2)}$$

Type A

$$u(n_1) = \sqrt{u_{\text{inj}}^2(n_1) + u_{\text{corr}}^2(n_1) + \left(\frac{\partial n_1}{\partial S_1}\right)^2 u_{\text{rep}}^2(S_1)}$$

gas

$$n_1 = \frac{PV_1}{RT}$$

liquid

$$n_1 = a_0 + a_1 S_1 + a_2 S_1^2$$

Calculation of the covariance matrix

$$\overline{S_{1,k}} = \frac{1}{N_k} \sum_{l=1}^{N_k} S_{1,k,l}$$

Type B

$$u_{\text{rep}}(S_1) = \frac{E_{\text{max}} S_1}{\sqrt{3}}$$

## □ Expanded uncertainty:

$$\text{➤ } U(P) = \textcircled{K} \sqrt{u_{\text{rep}}^2(P) + u_{\text{ref}}^2(P) + u_{\text{corr}}^2(P)}$$

**Coverage factor**

- $K=2$  defines an interval having a level of confidence of  $\approx 95\%$
- $K=3$  defines an interval having a level of confidence of  $\approx 99\%$

# Validation of data / Data Treatment



## □ Equations of state

- Data validation
- Pure compounds & mixtures

## □ Examples:

- Van der Waals type
  - ❖ VPT, SRK, PR

$$P = \frac{RT}{v-b} - \frac{a(T)}{v^2 + uv + wb^2}$$

- ❖ Alpha function: Mathias Copeman, Twu et al, etc...

- SAFT type: associating fluids

$$Z = 1 + Z^{\text{seg}} + Z^{\text{chain}} + Z^{\text{asso}}$$



# Pure compounds

## ○ Data validation: correlations

□ Antoine equation:  $\ln(P^{\text{sat}}) = A - \frac{B}{C + T}$

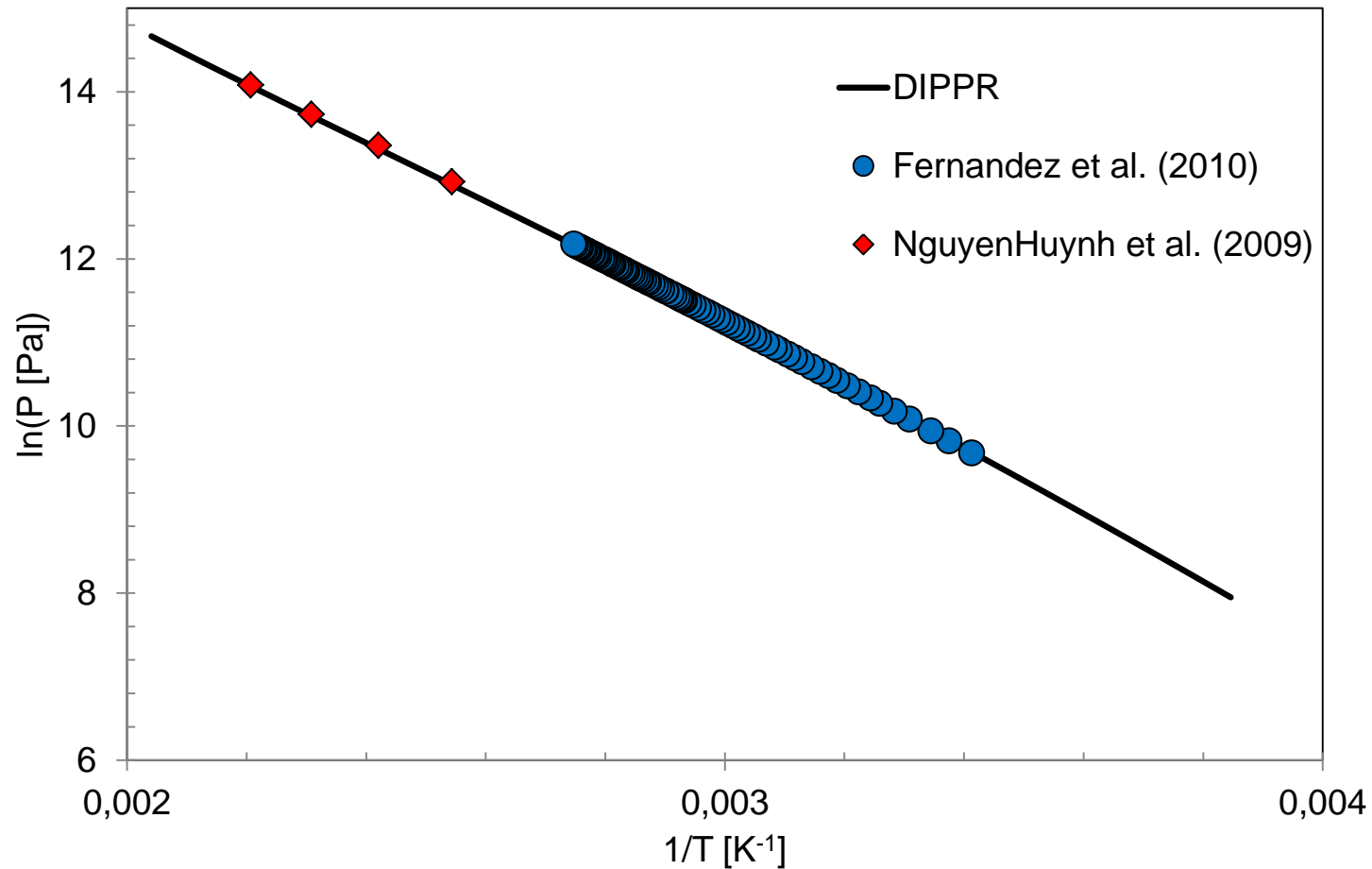
□ Frost Kalkwarf:  $\ln(P^{\text{sat}}) = A + \frac{B}{T} + C \ln(T) + D \frac{P^{\text{sat}}}{T^2}$

□ Wagner:  $\ln\left(\frac{P^{\text{sat}}}{P_c}\right) = \frac{T_c}{T} (A\tau + B\tau^{1.5} + C\tau^2 + D\tau^6) \quad \tau = 1 - \frac{T}{T_c}$

□ Cox:  $\ln\left(\frac{P^{\text{sat}}}{P_0}\right) = \left(1 - \frac{T_b}{T}\right) \exp(A_0 + A_1 T + A_2 T^2)$

□ DIPPR (Pa):  $\ln(P^{\text{sat}}) = A + \frac{B}{T} + C \ln(T) + DT^E$

## ○ Example: *n*-hexane



## □ Liquid density

- At atm. pressure: DIPPR [kmol.m<sup>-3</sup>]

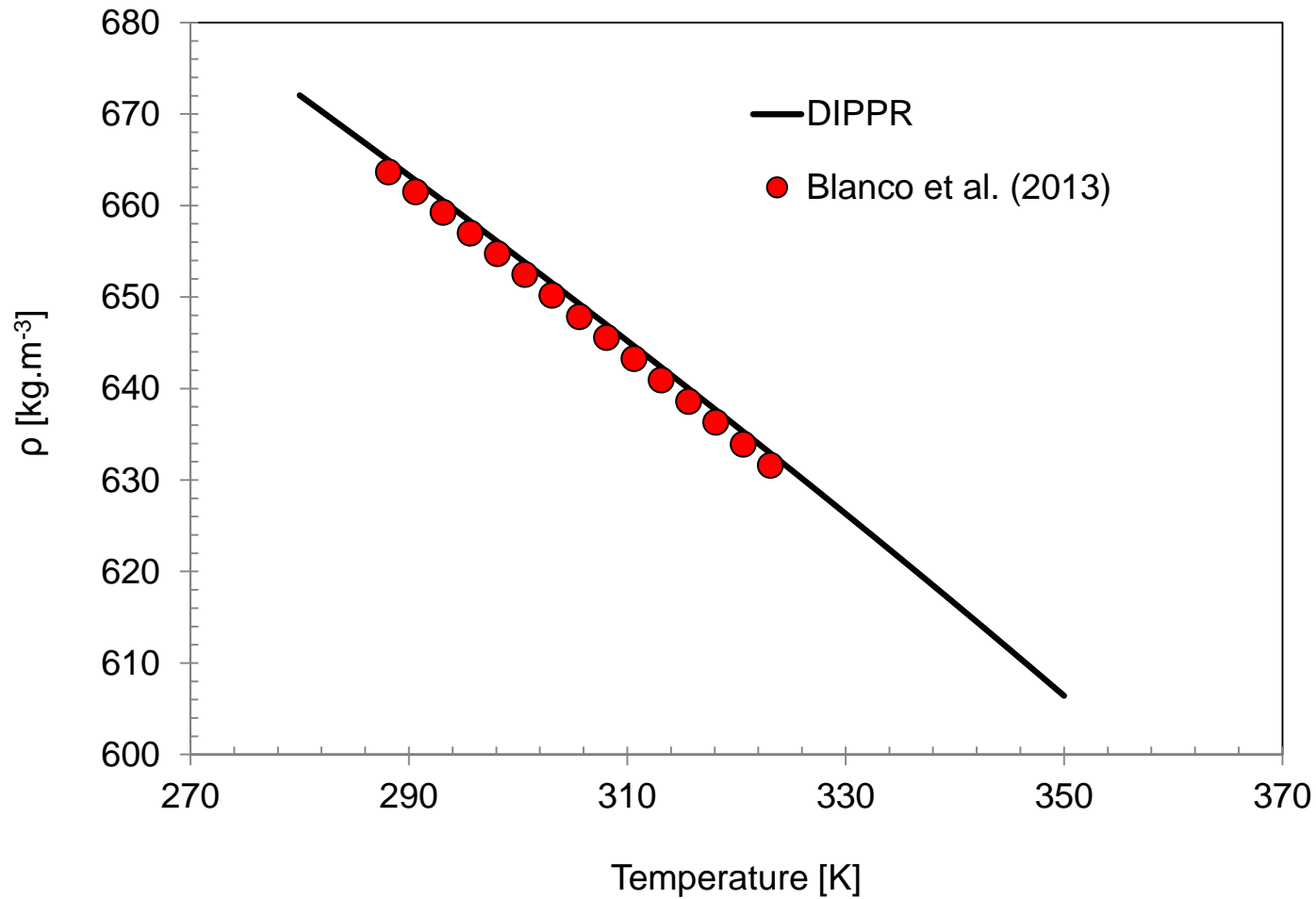
$$\rho = \frac{A}{B \left[ 1 + \left( 1 - \frac{T}{C} \right)^D \right]}$$

## □ Isothermal data

- Compression factor  $Z = \frac{P_v}{RT}$

- Virial equation (only for vapor phase)  $Z = 1 + B(T)\rho$

## ○ Example: *n*-hexane (liquid state)



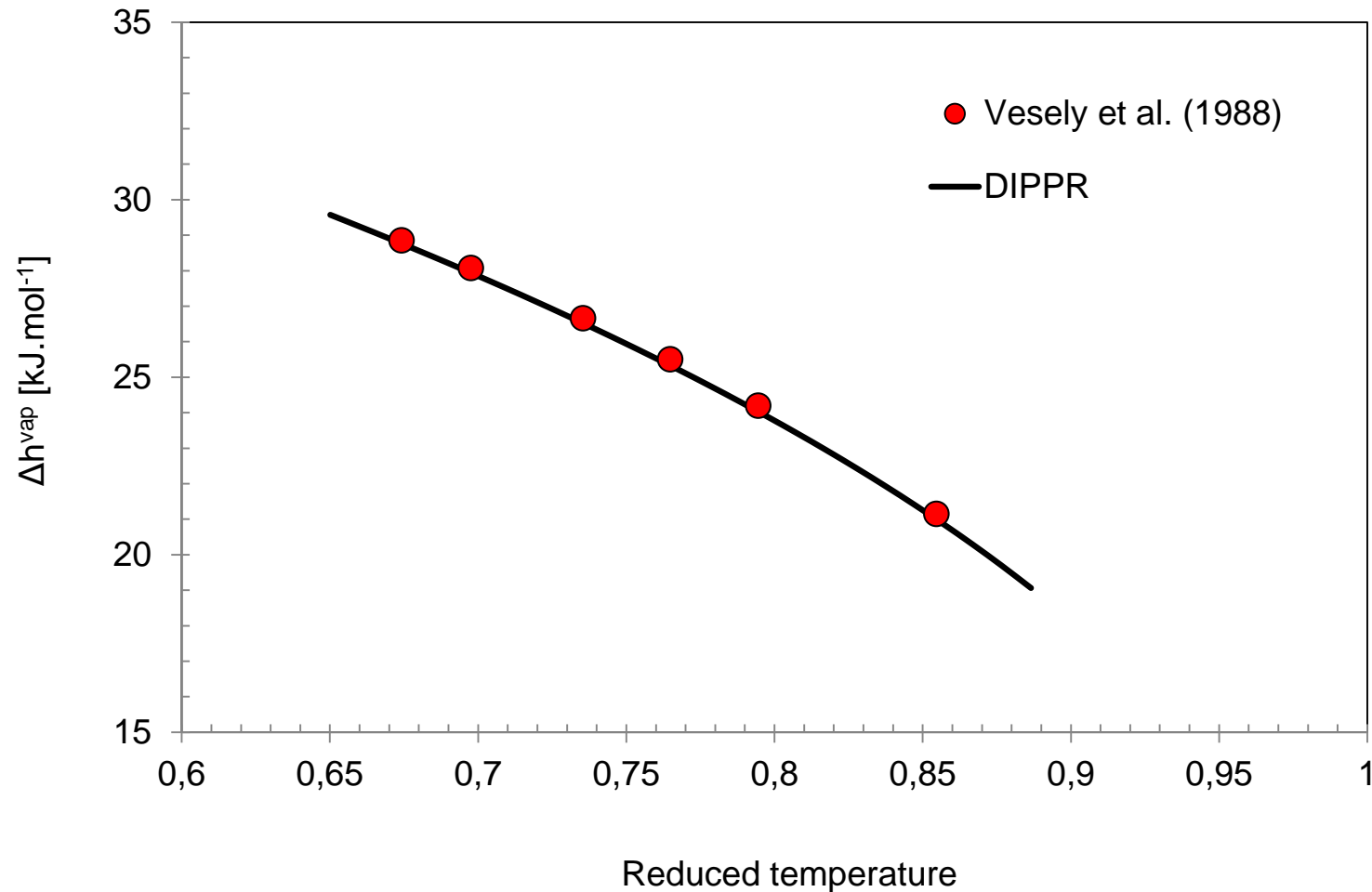
□ The Clapeyron equation 
$$\Delta h^{\text{sat}} = T(v^{\text{V,sat}} - v^{\text{L,sat}}) \frac{dP^{\text{sat}}}{dT}$$

□ DIPPR 
$$\Delta h = A(1 - T_r)^{B + CT_r + DT_r^2 + ET_r^3}$$

□ Watson (n=0.375) 
$$\frac{\Delta h_{T_2}}{\Delta h_{T_1}} = \left( \frac{1 - T_{r,2}}{1 - T_{r,1}} \right)^n$$

# Enthalpy of vaporization (2/2)

## ○ Example: *n*-hexane





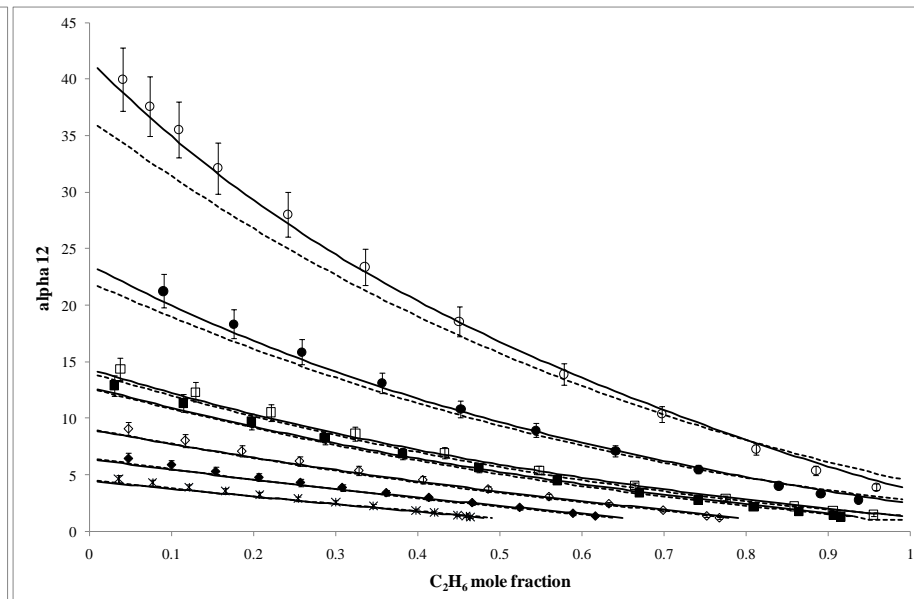
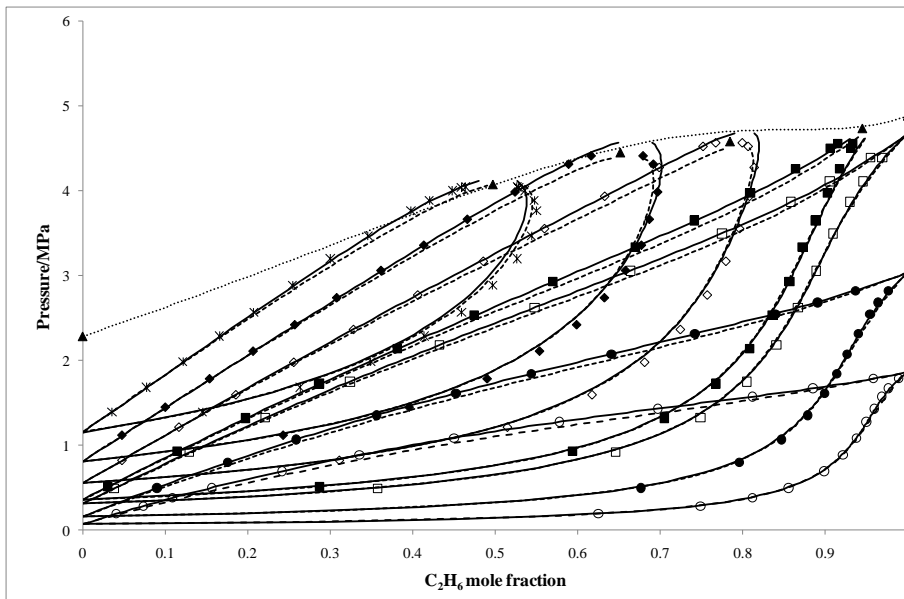


# Mixtures

## Graphic treatment: relative volatility

$$\alpha_{12} = \frac{y_1}{x_1} \cdot \frac{x_2}{y_2}$$

## Example: Ethane – C4F10



Solid lines: PR  
Dashed lines: PSRK

## □ Gibbs-Duhem equation

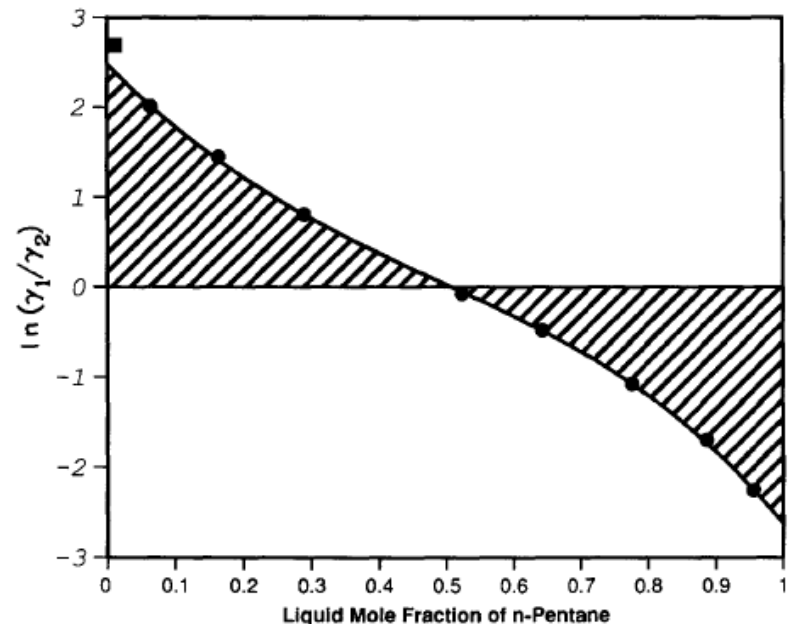
$$\sum_i x_i d \ln \gamma_i = -\frac{H^E dT}{RT^2} + \frac{V^E dP}{RT}$$

Thermodynamic model needed

## □ At equilibrium:

- Isothermal – isobaric form
- Entire composition range
- In low-pressure VLE

$$\int_0^1 \ln \frac{\gamma_1}{\gamma_2} dx_1 = 0$$



*Integral test for the n-pentane/methanol system.  
Jackson and Wilsak (1995)*

□ Usually:

Redlich – Kister polynomial correlation

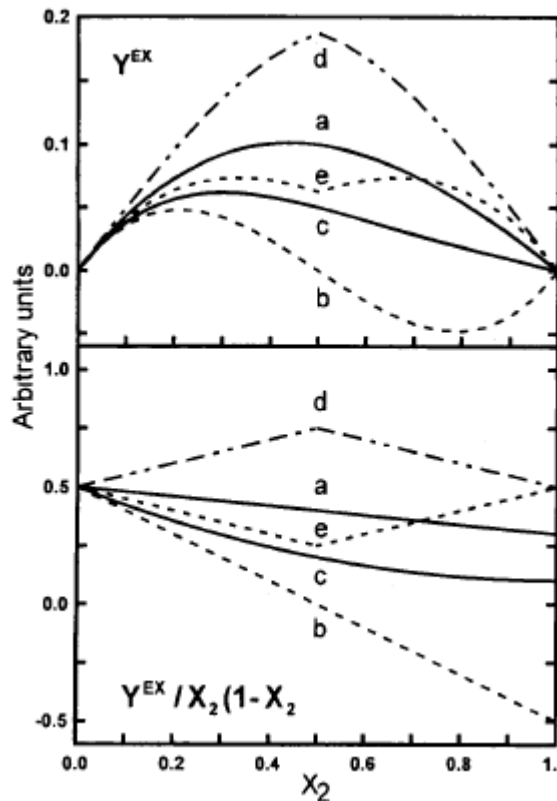
$$Y^E = x_1 x_2 \sum_i A_i (x_1 - x_2)^i$$

□ Problem: number of parameters

# Excess properties (2/2)

Desnoyers and Perron (1997)

$$\frac{Y^E}{x_1 x_2}$$



(a): Linear variation. Often mixtures of liquids of similar size and polarity

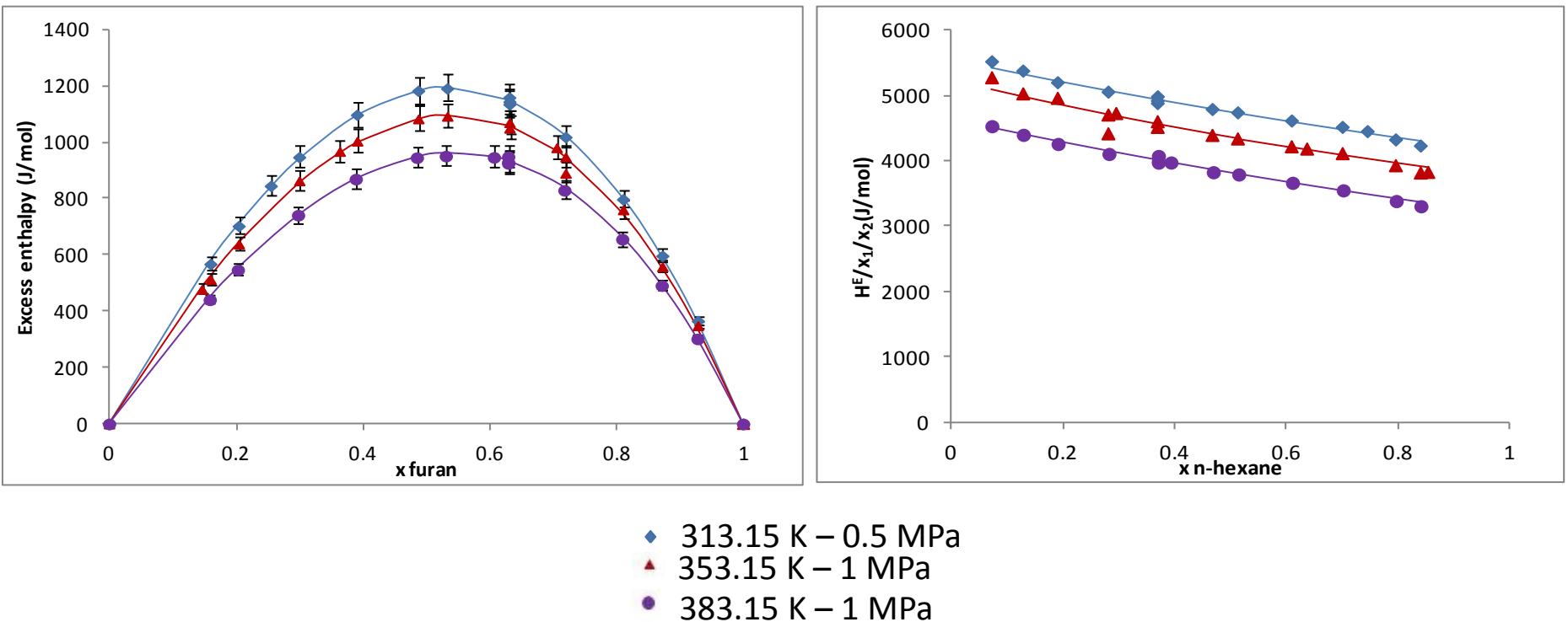
(b): Linear variation.  $Y_2^0 - Y_2^*$  opposite sign  $Y_1^0 - Y_1^*$

(c): Mixtures of liquids of different size and polarity

(d): Formation of complex (ex: mixtures of certain alcohols and amines)

(e): Similar to (d)

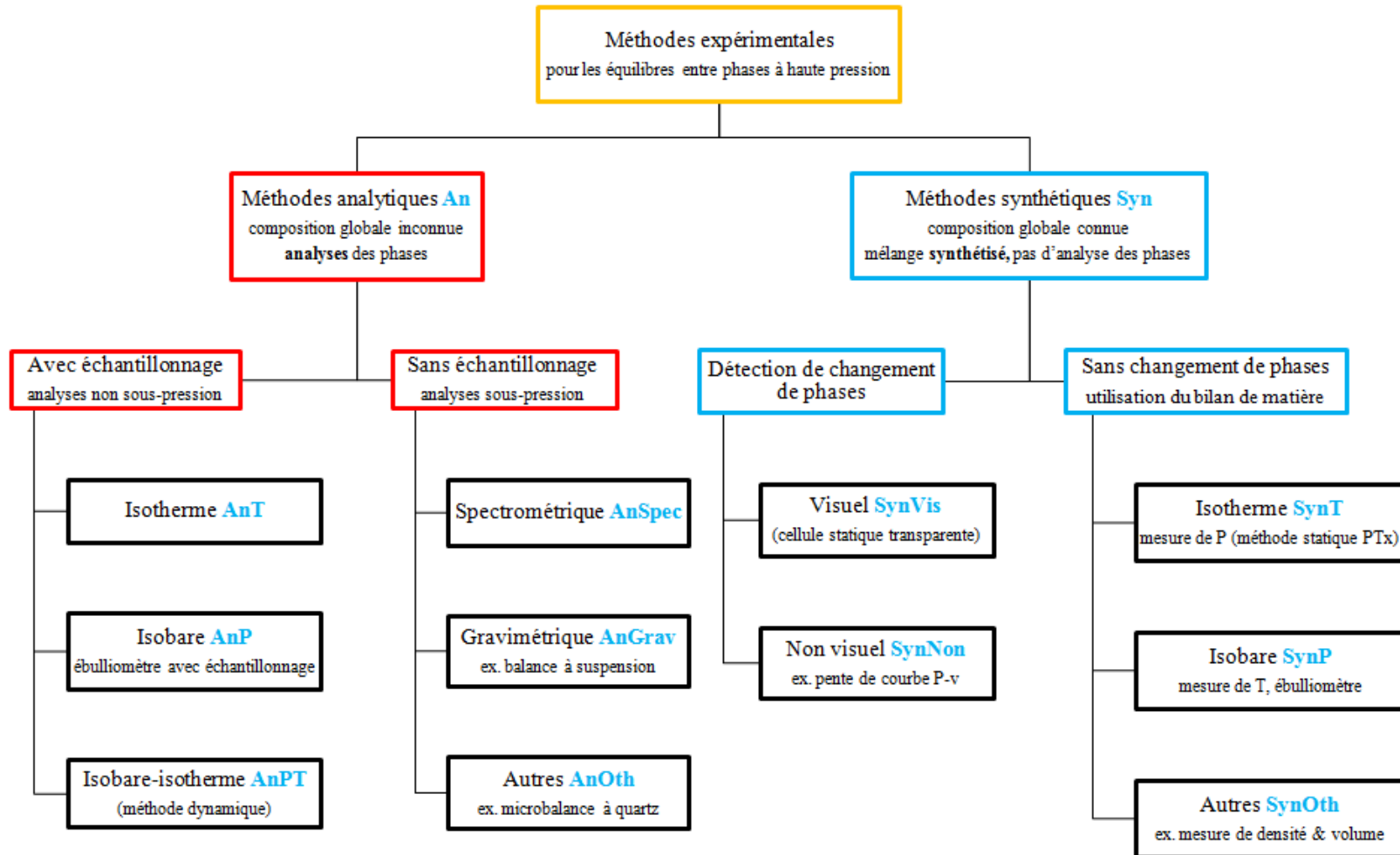
# Example: furan – *n*-hexane





# Conclusion

# Conclusion



*Experimental techniques for Phase Equilibria measurements at high pressures (Dohrn et al. 2010)*





# Thank you for your attention

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